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Stimulated Raman scattering instability of laser beam propagating through a collisional plasma in a self-focused filament

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Abstract : The stimulated Raman scattering instability is treated for laser beam propagating through a collisional plasma in a self-focused filament where nonlinear refraction due to the redistribution of the electron density caused by nonuniform ohmic heating of electron is balanced by diffraction divergence. The filament supports radially localized Langmuir waves A high power laser beam propagating through a collisional plasma breaks into high-intensity filaments of size $\sim c/\omega_p$ where c is the velocity of light in vacuum and ω_p is the plasma frequency Inside a filament, the laser undergoes stimulated Raman backscattering (B-SRS) Since the temperature inside a filament is higher and density is lower than those outside, the collisional damping rates of the decay waves are lowered and hence, the threshold power for B-SRS is reduced It is shown that self-focusing of perturbations in the incident laser beam can greatly enhance the growth rate of the instability This result may help to explain experimental observation of Raman scattering at nominal incident intensities well below the theoretical threshold

Keywords Stimulated Raman scattering instability, collisional plasma, self-focused filament

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1. Introduction

At short wavelengths, collisional effects considerably influence laser-plasma interaction. The nonlinear process of stimulated Raman scattering is seen to require laser power greater than a threshold value, determined by collisions. In several experiments, the observed values of threshold power are far below the values predicated theoretically [1-5]. Simon *et al* [6] invoked a two-state process to explain some of these results. The hot electrons produced *via* resonance absorption drive a Langmuir wave in the under dense region, *via*, two stream instability. The Langmuir wave couples with the pump to produce sidebands. Barr *et al* [7] have recently examined the SRS in the presence of a static sinusoidal density modulation, transverse to the axis of the pump laser radiation. Numerical solutions reveal that the tendency of Langmuir wave localization of the pump and consequent enhancement of its power density at the filament bottom, tend to enhance the growth rate. For parameters of interest, the later tendency may win over the former giving rise to an overall enhancement in the growth rate over its value in the uniform case. However, this calculation is numerical and the assumed density and intensity modulation are not self-consistent. Liu and Tripathi [8] have developed self-consistent theoretical model to obtain B-SRS growth rate in a cylindrical filament in a collisionless plasma. They take the size of the filament to correspond to the maximum linear spatial growth rate for the filamentation instability, with the result that for typical laser intensities, the SRS growth rate is only marginally changed form its value in the unfilamented incident beam. More recently, Afshar-Rad *et al* [9] have studied the evidence of stimulated Raman scattering occurring in laser filaments in long scale length plasmas.

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In this paper, we study B-SRS in a cylindrical filament where nonlinear refraction due to the redistribution of the electron is balanced by diffraction divergence. The filament supports radially localized Langmuir waves. The backscattered electromagnetic wave propagates in the density depleted channel primarily in the same mode as the pump wave; its width being comparable to the diameter of the filament. The coupled mode equations are solved by the first-order perturbation theory with ponderomotive nonlinearity.

In Section 2, we examine the self-consistent equilibrium of filaments. The coupled mode equations inside the filaments are derived in Section 3 for the Langmuir wave and the backscatter radiation. Employing perturbation theory, a nonlinear dispersion relation is derived and an expression for the growth rate is obtained. The results are discussed in Section 4.

2. Nature of filament

Consider an initially uniform laser wave propagating through a homogeneous plasma,

$$E_{0i} = \hat{y}A_0 \exp\left[-i(\omega_0 t - k_0 z)\right],$$

$$B_{0i} = ck_0 \times A_0/\omega_0 ,$$

$$k_0 = \epsilon_0^{1/2}\, \omega_0/c .$$

It gives rise to an osscillatory velocity of electrons

$$V_{0i} = \frac{eE_{0i}}{im\omega_0}\left(1 - i\frac{\nu}{\omega_r}\right) \tag{1}$$

where $-e$ and m are the electronic charge and mass. We first consider the filamentation instability by perturbing the amplitude $A_0 \rightarrow A_0 + A_1(r,z)$. The component of V_{0i} in phase with E_{0i} causes electron heating at the rate $-\frac{e}{2}E_{0i}^{*}V_{0i} = \frac{e^2 A_0 A_0^{*}\nu}{2m\omega_0^2}$, where ν is electron-ion collision frequency. In the steady state, this rate is balanced by the power loss *via* thermal conduction and collisions with ions and neutrals [10] :

$$-\nabla\left(\frac{\chi}{n}\nabla T_e\right) + \frac{3}{2}\delta\nu(T_e - T_1) = \frac{e^2\nu A_0 A_0^{*}}{2m\omega_0^2}, \tag{2}$$

where $\frac{\chi}{n} = \frac{v_{th}^2}{\nu}$, $\delta = 2\left(\frac{m}{m_1}\right)$ for electron-ion energy exchange collision, T_e and T_i being electron and ion temperatures and $v_{th} = \left(\frac{2T_e}{m}\right)^{1/2}$. For $\frac{\nu^2 r_0^2\delta}{v_{th}^2} > 1$, one may ignore the first term in eq. (2). Then

$$\frac{T_e - T_0}{T_0} = 2\alpha\left[A_0^2 + A_0\cdot(A_1 + A_1^{*})\right]. \tag{3}$$

Assuming quasi-neutrality and demanding the uniformity of plasma pressure $n(T_e + T_i) =$ constant, one obtains the modified density

$$n = n_0\left[1 - \frac{\alpha A_0(A_1 + A_1^{*})}{1 + \alpha A_0^2}\right], \tag{4}$$

where $\alpha = \frac{e^2}{6m\omega_0^2\delta T_0}$ and the effective permittivity of the plasma can be cast as

$$\epsilon = \epsilon_0 + \epsilon_2\, A_0\cdot(A_1 + A_1^{*}), \tag{5}$$

where $\epsilon_0 = 1 - \frac{\omega_p^2}{\omega_0^2}$, and $\epsilon_2 = \frac{\omega_p^2}{\omega_0^2}\frac{\alpha}{(1+\alpha A_0^2)}$,

where $\omega_p = \left(\frac{4\pi n_0 e^2}{m}\right)^{1/2}$ and no is the equilibrium plasma density. Using eqs. (1) and (5) in the wave equation and taking $\nabla\cdot(\epsilon E) = 0$ and $\frac{\partial \ln A_1}{\partial z} << k_0$, one obtains on linearization

$$2ik_0\frac{\partial A_1}{\partial z} + \frac{\partial^2 A_1}{\partial r^2} + \frac{1}{r}\frac{\partial A_1}{\partial r} + \frac{\omega_p^2}{c^2}\frac{\alpha A_0^2(A_1 + A_1^{*})}{(1+\alpha A_0^2)} = 0, \tag{6}$$

where $r = (x^2 + y^2)^{1/2}$ refers to a cylindrical polar coordinate. Expressing $A_1 = A_{1r} + iA_{1i}$ and separating real and imaginary parts, we get from eq. (7)

$$2k_0\frac{\partial A_{1i}}{\partial z} + \frac{\partial^2 A_{1r}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{1r}}{\partial r} = 0, \tag{7}$$

$$-2k_0\frac{\partial A_{1r}}{\partial z} + \frac{\partial^2 A_{1i}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{1i}}{\partial r} + \frac{2\omega_p^2}{c^2}\frac{\alpha A_0^2}{(1+\alpha A_0^2)}A_{1i} = 0$$

For $A_{1r}, A_{1i} \sim J_0(q_\perp r)e^{\Gamma z}$, eq. (7) straight way yields the spatial growth rate as

$$\Gamma = \frac{q_\perp}{2k_0}\left[-q_\perp^2 + 2k_0^2\frac{\epsilon_2}{\epsilon_0}A_0^2\right]^{1/2}. \tag{8}$$

The spatial growth maximizes to

$$\Gamma_{max} = \frac{\omega_p^2}{2k_0^2c^2}\frac{\alpha A_0^2}{1+\alpha A_0^2}, \tag{9}$$

at

$$q_{opt} = \frac{\omega_p}{c}\left(\frac{\alpha A_0^2}{1+\alpha A_0^2}\right)^{1/2}, \tag{10}$$

where $\alpha A_0^2 = \frac{1}{6}\frac{V_0^2}{c_s^2}$, $V_0 = \frac{e|A_0|}{m\omega_0}$, $c_s = \left(\frac{2T_0}{m_i}\right)^{1/2}$ and m_i is the mass of ion. The first zero of J_0 occurs at $q_\perp r = 2.4$. The amount of power tends to localize in maximally growing filament can be expressed as

$$P' = \frac{c}{8\pi}\pi r^2 A_0^2$$

$$= 4.3\,\frac{c^3 c_s^2 m^2\omega_0^2}{e^2\omega_p^2}\left(1+\alpha A_0^2\right). \tag{11}$$

Following Sodha *et al* [10], the temperature and density profile in the filament can be written as

$$T_e = T_0\left[1+2\alpha A_0^2\right]\left[1+2\alpha_1 E_0^2(r)\right],$$

$$n_0' = \frac{n_0}{\left[1+\alpha_1 E_0^2(r)\right]},$$

$$\nu = \nu_0\left(\frac{T_e}{T_0}\right)^{-3/2}\left(\frac{n_0'}{n_0}\right), \tag{12}$$

nd

$$\alpha_1 = \frac{\alpha}{1+2\alpha A_0^2},$$

where $E_0\,(r)$ is the total electric field of filament at r and ν_0 is collision frequency corresponding to n_0 and T_0. Expressing E_0 for cylindrically symmetric beam as $E_0 = A\ (r\ ,\ z)$ exp $\{i(\omega_0 t - k_0 z)\}$ and neglecting $\partial^2 A/\partial z^2$ which implies that the characteristic distance (in the z directions) of the intensity variation is much greater than the wavelength, the wave equation reduces to

$$2ik_0\frac{\partial A}{\partial z} + \nabla_\perp^2 A + \frac{\omega_p^2}{c^2}\left(1-\frac{n_0'}{n_0}\right)A = 0, \tag{13}$$

here

$$\nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r},$$

nd

$$A^2 = E_{00}^2\Big|_{r=0,z=0} e^{-r^2/r_0^2}.$$

Employing paraxial ray approximation[10], the radius of nonlinear steady state self-trapped cylindrical filament propagating through a homogeneous plasma, can be obtained from eq. (13) balancing diffraction and self-focusing terms [10]

$$R_d^2 = R_n^2, \tag{14}$$

where

$$R_d = k_0 r_0^2 \text{ and } R_n^2 = \frac{\omega_0^2}{\omega_p^2} r_0^2 \frac{\left(1+\alpha_1 E_{00}^2\right)^2}{\alpha_1 E_{00}^2}.$$

Eq. (14) determines the radius r_0 of a self-trapped filament as

$$r_0 = \frac{c}{\omega_p}\frac{\left(1+\alpha_1 E_{00}^2\right)}{\left(\alpha_1 E_{00}^2\right)^{1/2}}, \tag{15}$$

where E_{00} is the amplitude of the filament of radius r_0 in the nonlinear state, on the axis. The corresponding power in nonlinear steady state is

$$P = \frac{c}{8\pi}\pi r_0^2 E_{00}^2$$

$$= \frac{c^3}{8\omega_p^2}\frac{\left(1+\alpha_1 E_{00}^2\right)^2}{\alpha_1}.$$

Equating the power contained in the filament p to p' one obtains

$$\alpha_1 E_{00}^2 = \left[2.4\frac{\left(1+\alpha A_0^2\right)^{1/2}}{\left(1+2\alpha A_0^2\right)^{1/2}} - 1\right]. \tag{16}$$

Thus, the radius and field intensity in a self-trapped filament are dependent on the initial power density of the incident beam. The density, temperature and collision frequency variation near the axis of the filament can be obtained by expanding n_0', T_0 and ν around $r \equiv 0$ as given below :

$$n_0' = n_0^0\left(1+\frac{r^2}{a^2}\right), \tag{17}$$

$$T_e = T_0^0\left(1-\frac{r^2}{b^2}\right), \tag{18}$$

$$\nu = \nu_0^0\left(1+\frac{r^2}{d^2}\right), \tag{19}$$

where

$$a^2 = \frac{r_0^2\left(1+\alpha_1 E_{00}^2\right)}{\alpha_1 E_{00}^2},\quad b^2 = \frac{r_0^2\left(1+\alpha_1 E_{00}^2\right)}{2\alpha_1 E_{00}^2},$$

$$d^2 = \frac{2a^2 b^2}{3a^2+2b^2},$$

$$n_0^0 = \frac{n_0}{\left(1+\alpha_1 E_{00}^2\right)},\quad T_0^0 = T_0\left(1+2\alpha A_0^2\right)\left(1+2\alpha_1 E_{00}^2\right)$$

and $$\nu_0^0 = \nu_0\left(\frac{n_0^0}{n_0}\right)\left(\frac{T_0^0}{T_0}\right)^{-3/2}. \tag{20}$$

3. Coupled mode equations for B-SRS

Now, we consider the instability arising through the coupling of the pump wave (laser filament) obtained in the previous section with two small amplitude lower frequency waves in the filament:

an electromagnetic wave with frequency ω_1 and axial wave number $\boldsymbol{k}_1$ and a plasma wave with frequency ω and axial wave number $\boldsymbol{k}$, interacting with the pump wave with frequency ω_0 and axial wave number $\boldsymbol{k}_0$ (cf. Figure 1).

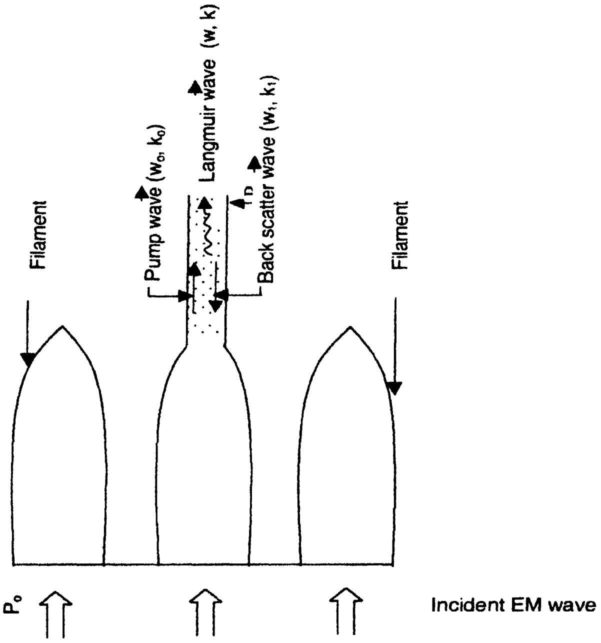

Figure 1. Schematic diagram of backward stimulated Raman scattering in a self-trapped laser filament

Consider the propagation of a laser filament in a density profile given by eq. (17) as

$$\boldsymbol{E}_0 = \boldsymbol{E}_0(r)e^{-i(\omega_0 t - k_0 z)}, \tag{21}$$

$$\boldsymbol{B}_0 \cong \frac{c\boldsymbol{k}_0 \times \boldsymbol{E}_0}{\omega_0}.$$

The pump wave produces an oscillatory electron velocity

$$\boldsymbol{v}_0 = \frac{e\boldsymbol{E}_0}{im\omega_0}$$

and parametrically excites a low frequency Langmuir wave with electrostatic potential

$$\phi = \phi(r)e^{-i(\omega t - kz)} \tag{22}$$

and a backscatter electromagnetic wave with electric and magnetic fields

$$\boldsymbol{E}_1 = \boldsymbol{E}_1(r)e^{-i(\omega_1 t - k_1 z)}, \quad \boldsymbol{B}_1 = \frac{c\boldsymbol{k}_1 + \boldsymbol{E}_1}{\omega_1}, \tag{23}$$

where $\boldsymbol{k}_1 = \boldsymbol{k} - \boldsymbol{k}_0$ and $\omega_1 = \omega - \omega_0$.

The sideband waves produce oscillatory electron velociti

$$\boldsymbol{v}_1 = \frac{e\boldsymbol{E}_1}{im\omega_1}, \qquad (2$$

and couple with pump wave to exert (ω, k) ponderomoti force on electrons, given by

$$F_p = e\,\vec{\nabla}\phi_p, \qquad (2$$

where $\phi_p = \dfrac{e\boldsymbol{E}_0 \cdot \boldsymbol{E}_1}{2m\omega_0\omega_1}$, driving the Langmuir wave

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \left[\left\{\frac{\left(\omega^2 - \omega_{po}^2 - k^2 v_{tho}^2 + i\omega v_0^0\right)}{v_{tho}^2}\right\} - \left\{r^2\frac{\omega_{po}^2}{a^2 v_{tho}^2}\left(1 - \frac{i v_0^0 \omega\, a^2}{\omega_{po}^2\, c_1^2}\right)\right\}\right]\phi$$

$$= -\frac{\omega^2|v_0|}{2\omega_0 v_{th}^2}, \qquad (2$$

where $v_{tho} = \left(\dfrac{2T_0^0}{m}\right)^{1/2}$, $v_0 = v_{osc}\exp\left(-\dfrac{r^2}{2a^2}\right)$,

$$v_{osc} = \frac{eE_{00}}{m\,\omega_0},\quad c_1^2 = \frac{b^2 d^2}{b^2 + d^2},\quad \omega_{po} = \left(\frac{4\pi n_0^0 e^2}{m}\right)^{1/2},$$

and we have assumed only collisional damping. The densi perturbation

$$n(\omega, \boldsymbol{k}) \cong k^2\phi/4\pi e \qquad (2$$

couples with v_o to produce a nonlinear current density

$$J_1^{NL} = -ne\frac{\boldsymbol{v}_0}{2}. \qquad (2$$

The nonlinear current density at the side band frequen can be written as

$$J_1 = -n_0^0 e\boldsymbol{v}_1 - \frac{1}{2}ne\boldsymbol{v}_0. \qquad (2$$

Using eq. (29) in the wave equation, we get

$$\frac{\partial^2 E_1}{\partial r^2} + \frac{1}{r}\frac{\partial E_1}{\partial r} + \left(\frac{\omega_1^2 - \omega_{po}^2 - k_1^2 c^2}{c^2} - \frac{\omega_{po}^2}{a^2}\frac{r^2}{c^2}\right)E_1$$

$$= -\frac{k^2\omega_0|v_0|}{2c^2}\phi. \qquad (3$$

It is considered that the sideband wave is not affected by Landau damping. However, it may suffer damping due to collisions. In this case, eq. (30) is modified to

$$\frac{\partial^2 E_1}{\partial r^2}+\frac{1}{r}\frac{\partial E_1}{\partial r}+\left[\left\{\frac{\omega_1^2-\omega_{po}^2\left(1-i\frac{\nu_0^0}{\omega_1}\right)-k_1^2c^2}{c^2}\right\}-\left\{\frac{\omega_{po}^2}{a^2}\frac{r^2}{c^2}\left(1-\frac{i\nu_0^0}{\omega_1}\frac{a^2}{d^2}\right)\right\}\right]E_1$$

$$=-\frac{k^2\omega_0|\nu_0|}{2c^2}\phi\,. \quad (31)$$

The electromagnetic and plasma normal modes satisfy eqs. 7) and (31) in the absence of nonlinear coupling, *i.e.*

$$\frac{\partial^2 E_1}{\partial r^2}+\frac{1}{r}\frac{\partial E_1}{\partial r}+k_{1\perp}^2E_1-\alpha_1^{-4}r^2E_1=0\,,$$

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+k_\perp^2\phi-\alpha^{-4}r^2\phi=0\,, \quad (32)$$

here

$$k_\perp^2=\frac{\omega^2-\omega_{po}^2-k^2v_{tho}^2+i\omega\nu_0^0}{v_{tho}^2}\,,$$

$$k_{1\perp}^2=\frac{\omega^2-\omega_{po}^2-k_1^2c^2+i\omega_{po}^2\frac{\nu_0^0}{\omega_1}}{c^2}\,,$$

$$\alpha^{-4}=\frac{\omega_{po}^2}{a^2v_{tho}^2}\left(1-\frac{i\nu_0^0\omega}{\omega_{po}^2}\frac{a^2}{c_1^2}\right),$$

$$\alpha_1^{-4}=\frac{\omega_{po}^2}{a^2c^2}\left(1-\frac{i\nu_0}{\omega_1}\frac{a^2}{d^2}\right).$$

(32) have well-behaved solutions [11] :

$$k_\perp^2=2(\ell+1)\frac{\omega_{po}}{av_{tho}}\left(1-\frac{i\nu_0^0\omega}{\omega_{po}^2}\frac{a^2}{c_1^2}\right)^{1/2}, \quad (33)$$

$$k_{1\perp}^2=2(m+1)\frac{\omega_{po}}{ac}\left(1-\frac{i\nu_0^0}{\omega_1}\frac{a^2}{d^2}\right)^{1/2}, \quad (34)$$

$$\phi=\phi_l=\Gamma_l L_l\left(\frac{r^2}{b_1^2}\right)\exp\left(-\frac{r^2}{2b_1^2}\right)\exp\left(\frac{ir^2}{4b_1^2}\frac{\nu_0^0\omega}{\omega_{po}^2}\frac{a^2}{c_1^2}\right), \quad (35)$$

$$E_1=E_{1m}=\Gamma_m L_m\left(\frac{r^2}{b_2^2}\right)\exp\left(-\frac{r^2}{2b_2^2}\right)\exp\left(\frac{ir^2}{4b_2^2}\frac{\nu_0^0}{\omega_1}\frac{a^2}{d^2}\right), \quad (36)$$

where $b_1=\left(\frac{av_{tho}}{\omega_{po}}\right)^{1/2}$, $b_2=\left(\frac{ca}{\omega_{po}}\right)^{1/2}$,

$$L_l(\xi)=e^{\xi}\frac{d'}{d\xi'}\left(\xi'e^{-\xi}\right),$$

$\ell=0,1,2,\ldots\ldots\ldots;m=0,1,2,\ldots\ldots\ldots\ldots;$

and Γ_l and Γ_m are normalization constants. Since the pump field (hence ν_0) scales as $\exp\left(-\frac{r^2}{2a^2}\right)$, the most unstable backscatter mode would correspond to $m=0$. In the presence of nonlinear coupling terms, one could express ϕ in terms of an orthogonal set of wavefunctions ϕ_l, where as E_1 can be taken to be the dominant mode:

$$\phi=\sum_l s_l\phi_l\,,\ \text{and}\ E_1=TE_{10}\,. \quad (37)$$

Using eqs. (37), (26) and (31), multiplying the resulting equation by ϕ_l and E_{10}, respectively and one gets

$$\left[\left\{\frac{\omega^2-\omega_{po}^2-k^2v_{tho}^2+i\omega\nu_0^0}{v_{tho}^2}\right\}-\left\{2(\ell+1)\frac{\omega_{po}}{av_{tho}}\left(1-\frac{1}{2}\frac{\nu_0^0\omega}{\omega_{po}^2}\frac{a^2}{c_1^2}\right)\right\}\right]S_l$$

$$=-\frac{\omega^2T}{2\omega_0v_{tho}^2}\int rdr|\nu_0|\phi_lE_{10}\left(1+\frac{r^2}{b^2}\right), \quad (38)$$

$$\left[\left\{\frac{\omega_1^2-\omega_{po}^2\left(1-\frac{i\nu_0^0}{\omega_1}\right)-k_1^2c^2}{c^2}\right\}-\left\{\frac{2\omega_{po}}{ac}\left(1-\frac{1}{2}\frac{i\nu_0^0}{\omega_1}\frac{a^2}{d^2}\right)\right\}\right]T$$

$$=-\frac{k^2\omega_0}{2c^2}\Sigma_l S_l\int rdr|\nu_0|\phi_lE_{10}\,, \quad (39)$$

leading to a nonlinear dispersion relation

$$\left[\left\{\omega_1^2-\omega_{po}^2\left(1-\frac{i\nu_0^0}{\omega_1}\right)\right\}-\left\{k_1^2+\frac{2\omega_{po}}{ac}\left(1-\frac{1}{2}\frac{i\nu_0^0}{\omega_1}\frac{a^2}{d^2}\right)\right\}c^2\right]$$

$$=4\gamma_0^2\omega\omega_0\times\Sigma_l$$

$$\frac{\left(I_1^2+\dfrac{I_1 I_1(1)}{b^2}\right)}{\left[\left(\omega^2-\omega_{po}^2+i\omega v_0^0\right)-\left\{k^2+\dfrac{(2\ell+1)\omega_{p0}}{a v_{tho}}\left(1-\dfrac{1}{2}\dfrac{v_0^0\omega a^2}{\omega_{po}^2 c_1^2}\right)\right\}v_{tho}^2\right]}, \tag{40}$$

where

$$I_1=\int_0^\infty r\,dr\,\phi_1\,E_{l0}\exp\left(-\frac{r^2}{2a^2}\right),$$

$$I_1(1)=\int_0^\infty r^3 dr\,\phi_1\,E_{l0}\exp\left(-\frac{r^2}{2a^2}\right), \tag{41}$$

$\gamma_0=\frac{1}{4}(kv_{osc})(\omega/\omega_0)^{1/2}$ is uniform medium growth rate and we have used $v_0\equiv v_{osc}e^{-r^2/2a^2}$. Since ϕ_1 is localized in a narrow region around $r\le b_1 << a$, I_1 and I_1 (1) may be simplified to become

$$I_1\equiv\frac{\sqrt{2}}{a}\int_0^\infty r\,dr\,\phi_1 \text{ and } I_1(1)\equiv\frac{\sqrt{2}}{a}\int_0^\infty r^3 dr\,\phi_1\,. \tag{42}$$

In the presence of the pump wave, let $\omega=\omega_r+i\gamma$ where ω_r is the frequency at which the factor in eq. (40) exactly vanish then the nonlinear dispersion relation gives the growth rate as

$$(\gamma+\gamma_s)(\gamma+\gamma_e)=\gamma_{00}^2\,, \tag{43}$$

where γ_e an γ_s are the amplitude damping rates for Langmuir and backscatter wave and γ_{00} is the growth rate in the absence of damping. For $\ell=0$, mode, γ_e, γ_s and γ_{00} can be expressed as :

$$\gamma_e=\frac{v_0^0}{2}\left(1+\frac{v_{tho}}{a\omega_{po}}\frac{a^2}{c_1^2}\right), \tag{44}$$

$$\gamma_s=\frac{1}{2}\frac{v_0^0\omega_{po}^2}{\omega_0^2}\left(1+\frac{a^2}{d^2}\frac{c}{a\omega_{po}}\right), \tag{45}$$

and

$$\gamma_\infty=\gamma_0 I_\ell\left(1+\frac{1_\ell I_\ell(1)}{b^2 I_\ell^2}\right)^{1/2}. \tag{46}$$

Damping of the unstable waves introduces a threshold intensity for instability generation. The threshold condition due to damping is

$$\gamma_\infty\ge\sqrt{\gamma_e\gamma_s}\,. \tag{47}$$

Considering backscatter for $\frac{\omega_{pe}}{\omega_0}<<\frac{1}{2}$ and assuming only collisional damping, eqs. (44), (45), (46), and (47) give threshold value of $\frac{v_{osc}}{c^2}$ as

$$P_{th}''=\left(\frac{v_{osc}}{c^2}\right)^2_{SRS-th}=\frac{1}{4}\left(\frac{\omega_{p0}}{\omega_0}\right)^2\cdot\frac{v_0^{0^2}}{\omega_0\omega_{p0}}\frac{yz}{x}, \tag{48}$$

where $y=\left(1+\frac{v_{tho}}{a\omega_{po}}\frac{a^2}{c_1^2}\right)$, $z=\left(1+\frac{c}{a\omega_{po}}\frac{a^2}{d^2}\right)$,

and $x=\frac{b_1^2}{b_2^2}\left(1+\frac{2b_1^2}{b^2}\right)$. (49)

This threshold intensity can be quite low. One may mention that the threshold condition for B-SRS when background plasma and intensity of laser beam is uniform, is written as

$$P_{th}'=\left(\frac{V_0}{c}\right)^2_{SRS-th}=\frac{1}{4}\left(\frac{\omega_p^2}{\omega_0^2}\right)\frac{v_0^2}{\omega_0\omega_p}\,. \tag{50}$$

Substituting eqs. (44), (45) and (46) into eq. (43), the maximum growth rate can be expressed as

$$\gamma=\gamma_0\frac{2b_1}{b_2}\left(1+\frac{2b_1^2}{b^2}\right). \tag{51}$$

It is much more worthwhile to compare this growth rate with the one (γ_0') when the lower wave is uniform. Since $\gamma_0'=\frac{1}{4}kv_0\left(\frac{\omega}{\omega_0}\right)^{1/2}$, one obtains

$$\frac{\gamma}{\gamma_0'}\cong\frac{v_{osc}}{v_0}2\left(\frac{v_{tho}}{c}\right)^{1/2}\left(1+\frac{2b_1^2}{b^2}\right). \tag{52}$$

For $\frac{v_0}{\omega_0}=0.02$, $\frac{\omega_p^2}{\omega_0^2}=0.005$, $T_o=100$ eV, $\omega_0=7.2\times 10$ rad sec^{-1} ($\lambda=0.26\mu m$), $c_s=2\times10^7$ cm sec^{-1} and $z=69$ (gold plasma) P_{th}' and P_{th}'' turns out to be 7×10^{-6} and 1.4×10 respectively. For $T_o=100$ eV, and $\alpha A_0^2=3$, the ratio $\frac{\gamma}{\gamma_0'}$ is of the order of unity and the radius of the self-trapped filament turns out to be 2 μm. Threshold intensities corresponding to P_{th}' and P_{th}'' are ~ 1.5×10^{14} W cm^{-2} and 2×10^{13} W cm^{-2}

The process of B-SRS in a filament is aided by the enhancement of power density over its initial value but it is inhibited by the localization of Langmuir wave and hence of the interaction region. It is observed that the power density inside the filament is much greater than the initial power density of the laser beam. Hence, the enhanced intensity in laser filament reduces collisional damping of backscatter light wave, diminishing the threshold power for B-SRS. The onset of B-SRS is strongly correlated with intensity threshold of the filamentation instability. Figure 2 shows the evidence of B-SRS occurring in laser filaments. It is seen that P_{th}'' is decreasing function of temperature. The growth rate of perturbation increases with temperature.

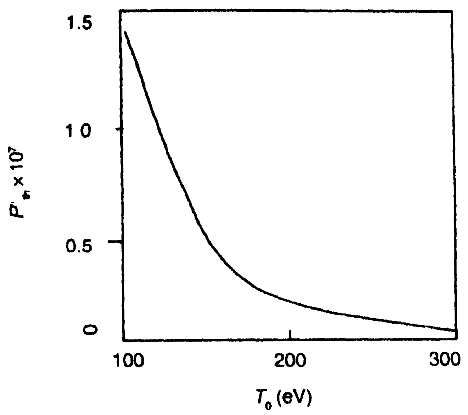

Figure 2. Variation of dimensionless threshold power $P_{th}''\left(=(v_{ox}/c)^2_{SRS-TH}\right)$ for B-SRS in a laser filament with initial temperature (T_0) for v_0/ω_0 = 0.02, ω_p^2/ω_0^2 = 0.005, T_0 = 100 eV, c_s = 2 × 10^7 cm sec^{-1}.

4. Conclusion

The process of stimulated Raman backscattering in a filament is aided by the enhancement of power density over its initial value. The higher temperature and lower densities occurring locally inside the filaments lower the collisional damping rates of the decay wave; hence, the threshold power for B-SRS is reduced. P_{th}'' is a decreasing function of initial temperature. Threshold power density required to onset the B-SRS in cylindrical filament is smaller or equal to the threshold for filamentation instability. At lower intensities and higher temperatures, filamentation tends to enhance the growth rate. Threshold power density decreases with the frequency of the pump wave. The ratio of growth rate in a filament to that in a uniform beam (and uniform plasma) is of the order of unity, implying that filamentation doesn't have much effect on B-SRS.

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